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In order that N may be the sum of three triangular numbers, t_n, t_p, t_r, it is necessary and sufficient that $8N+3=(2n+1)^2+(2p+1)^2+(2r+1)^2$, i. e., be the sum if three odd squares.

Ex. 3. Let $N=1878$; $8N+3=15027=121^2+19^2+5^2$. $\therefore 1878 = t_{60}+t_9+t_2$.

Since every odd square number, as we have seen above, is of the form $8t_n+1$, the sum of any three odd squares must be of the form $8N+3$. But of the converse proposition, that $8N+3$ is always the sum of three odd squares, I have been unable to obtain a demonstration, though the fact can always be verified upon trial. The greatest square $(2n+1)^2$ is usually the greatest odd square in $8N+3$, or the next odd square below; the second square in the sum is, in like manner, usually the greatest odd square in $8N+3-(2n+1)^2$.

Perhaps some reader of the ANALYST can find a demonstration of one or both of the following propositions, in which N, x, y, z, may be integers or zero.

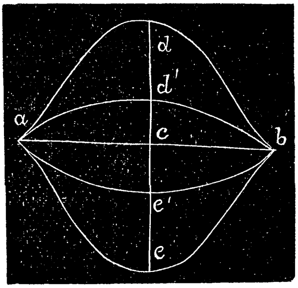
$$8N+3=(2x+1)^2+(2y+1)^2+(2z+1)^2. \tag{1}$$

$$4N+1=(2x+1)^2+4y^2 \quad +4z^2. \tag{2}$$

NOTE ON THE SOLUTION OF PROB. 217, BY THE EDITOR.—When the note at the foot of page 155 was written, we supposed that Prof. Johnson's solution necessarily gave one branch of the curve on each side of the axis of x, as it was supposed that the triangles were only drawn on *one side* of the hypothenuse; and the integration we gave at page 156 was made with that understanding.

By drawing the triangles on *both sides* of the hypothenuse, and describing squares on the *outside* of the legs *above* the hypothenuse, and on the *inside* of the legs *below* the hypothenuse, both branches of the locus will be *above* the axis of x, and consequently they will contain the area $(\frac{16}{3}\sqrt{3}-9)\pi a^2$, as found by Prof. Johnson. On the other hand, if we consider only the triangles that can be drawn on one side of the hypothenuse, and describe the squares on both sides of the legs, we obtain two branches respectively equivalent to those found by Prof. Johnson, but one lying above and the other below the axis of x, and consequently enclosing an area $=\frac{16}{9}\sqrt{3}\,.\pi a^2-4a^2$, as given by us at page 156.

Both branches as thus found are represented by equation (4), page 155, but one, the outer branch, is excluded by the wording of the question, and hence Messrs. Seitz, Heaton and Baker, by describing the squares only on the outside of the legs, find two equal branches, one above and the other below the axis of x, which enclose an area $=(9-\frac{32}{9}\sqrt{3})\pi a^2-8a^2$.



Equation (4) may be written

$$y = \pm \frac{a^2 - x^2}{2a \pm \sqrt{(a^2 - x^2)}},$$

and hence represents four branches, as in the marginal diagram. *a d′ b d a* is the area as found by Prof. Johnson; *a e′ b d a* is the area as found by us, and *a e′ b d′ a*, is the area as found by Messrs. Seitz, Heaton and Baker.

Prof. Johnson writes as follows:—

"The expression given on page 156, line 2, is the value of

$$\int \frac{d\theta}{2+\cos\theta} \text{ and not of } \int \frac{d\theta}{2+\sin\theta},$$

but the result is not affected, since, if we put $\theta = \theta' - \frac{1}{2}\pi$, we have

$$\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} = \int_{\frac{1}{2}\pi}^{\frac{5}{2}\pi} \frac{d\theta'}{2+\sin\theta'} = \int_0^{2\pi} \frac{d\theta}{2+\sin\theta}.\text{"}$$

NOTE ON THE POLYNOMIAL THEOREM.

BY PROF. W. W. JOHNSON.

THE Binomial Theorem may be written in the form

$$\frac{(a+b)^n}{n!} = \frac{a^n}{n!} + \frac{a^{n-1}}{n-1!} \cdot \frac{b}{1} + \frac{a^{n-2}}{n-2!} \cdot \frac{b^2}{2!} + \ldots$$

and if we put ${}_ra = \frac{a^r}{r!}$ (we might call ${}_ra$ the rth *pyramid* of a, since ${}_2a$ is the area of a triangle whose base and altitude are a, and ${}_3a$, the volume of a pyramid whose base is ${}_2a$ and altitude a), this becomes

$${}_n(a+b) = \Sigma \,.\, {}_ra\,{}_sb,$$

where $r + s = n$ and r admits of all values from 0 to n inclusive. It follows at once that

$${}_n(a+b+c) = \Sigma \,.\, {}_p(a+b)\,{}_tc = \Sigma \,.\, {}_ra\,{}_sb\,{}_tc,$$

where $r+s+t = n$; and in general

$${}_n(a+b+c+ \ldots) = \Sigma \,.\, {}_ra\,{}_sb\,{}_tc \ldots$$

where $r+s+t+ \ldots = n$. This last equation is a form of the multinomial theorem.